Scaling analysis of the magnetoresistance in $Ga_{1-x}Mn_xAs$: Evidence for strong fluctuation and interaction effects

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(Dated: April 16, 2008)

We compare experimental resistivity data on $Ga_{1-x}Mn_xAs$ films with theoretical calculations using a scaling theory for strongly disordered ferromagnets. All characteristic features of the temperature dependence of the resistivity can be quantitatively understood through this approach as originating from the close vicinity of the metal-insulator transition. In particular, we find that the magnetic field induced changes in resistance cannot be explained within a mean-field treatment of the magnetic state, and that accounting for thermal fluctuations is crucial for a quantitative analysis. Similarly, while the non-interacting scaling theory is in reasonable agreement with the data, we find clear evidence in favor of interaction effects at low temperatures.

PACS numbers: 72.20.i, 72.80.r, 75.50.Pp, 72.20.My

Over the past decade, $Ga_{1-x}Mn_xAs$ has been the most extensively studied ferromagnetic semiconductor because it can be used in proof-of-concept spintronic devices [1]. In particular, the carrier-mediated ferromagnetism in this material makes it attractive for device integration with the technologically mature III-V semiconductors such as GaAs, where bandgap engineering allows systematic modulation of the carrier density in heterostructure devices. Furthermore, we note that an all-electrical III-V semiconductor spintronic device has been recently demonstrated [2]. It is important within this general context to develop a fundamental understanding of the interplay between carrier transport and magnetism in $Ga_{1-x}Mn_xAs$. One of the basic but least understood properties of $Ga_{1-x}Mn_xAs$ is the temperature and magnetic field dependence of its resistivity [3].

For typical $Ga_{1-x}Mn_xAs$ samples the resistivity increases with decreasing temperature above the Curie temperature, T_C, but then it suddenly drops below T_C thereby resulting in a resistivity peak at T_C, which then gradually broadens and gets shifted to higher temperatures upon application of a magnetic field (see Fig. 1). In "high quality samples", the aforementioned increase is less pronounced and one observes a broad shoulder rather then a resistivity peak. The resistivity also shows another upturn at much lower temperatures. There have been a number of attempts to explain the resistivity peak so far [3], which invoked, e.g., scattering by critical fluctuations [4], the formation of magnetic polarons [5, 6], 'dynamical' mean field calculations [7], or the interplay with universal conductance fluctuations [8]. Although these theoretical approaches have been quite successful in addressing a particular range or qualitative aspect of the data, however, a theoretical framework that could quantitatively explain all characteristic features observed in $Ga_{1-x}Mn_xAs$ has not been available so far. Besides the simplicity of our model and the clear set of reasonable parameter choices we make, the attractive feature of the approach presented in this paper stems from the fact that it is capable of *quantitatively* account for all distinct features of the experimental data: (i) the gradually increasing resistance as the temperature is lowered towards T_C; (ii) the pronounced peak precisely at T_C; (iii) the upturn in resistance at low temperatures, together with the finite resistance intercept for metallic samples; (iv) the precise amount with which an external magnetic field depletes the resistance peak at T_C and shifts it towards higher temperatures; (v) the remarkable "non-crossing" constraint of the experimental data, a constraint that some other theories fail: the resistance curves that belong to different values of the external field do *not* cross.

In this Letter, we present a theoretical analysis of the temperature dependent resistivity of $Ga_{1-x}Mn_xAs$ and show good agreement with a range of measured samples despite of the simple assumptions we make. First of all, we remark that the features mentioned above are reminiscent of localization effects interplaying with magnetism. In fact, most $III-V$ ferromagnetic semiconductors are very bad conductors because the charged dopants (Mn for $Ga_{1-x}Mn_xAs$) introduce very large disorder [9] and can lead even to the formation of an impurity band [10]. As we show later, even for annealed and relatively highly-doped samples, a naive estimate gives $k_F l \sim 0.3$, with k_F the Fermi momentum and l the mean free path. This value clearly hints towards the importance of disorder, and suggests that it is necessary to beyond the popular free electron picture frequently used in the literature [11], . [12]. Strikingly similar resistivity anomalies have been also observed in other types of magnetic semiconductors [13], as well as some manganites [14, 15], with

various semi-phenomenological frameworks available for explaining these phenomena in terms of localization theory [15, 16, 17]. However, these approaches focus explicitly on the localized phase. Such a starting point is not suitable for a fairly large class of experimentally measured $Ga_{1-x}Mn_xAs$ samples that are not insulators, but very poor metals, close to the localization transition.

In Ref. [18], we developed a scaling theory of magnetoresistance, wherein all the characteristic features of the resistivity anomalies appear naturally. This previous work primarily focused on the localized phase, where Mott's variable range hopping formula can be used to compute the resistivity. The purpose of the present paper is to extend this theory to metallic samples, and show that it *qualitatively* and *quantitatively* explains detailed aspects of the magneto-transport properties. To understand the properties of $Ga_{1-x}Mn_xAs$ we must take into account electron-electron interaction too. Nevertheless, as explained later, the concept of universal one-parameter scaling carries over to this material due to the large intrinsic spin-orbit coupling of $Ga_{1-x}Mn_xAs$ and the presence of magnetic moments.

To test the theoretical approach, we have measured a series of annealed and unannealed $Ga_{1-x}Mn_xAs$ samples grown by molecular beam epitaxy in which the Mn concentration (x) was systematically varied between 0.0135 and 0.067. The temperature dependent resistivity data shown in Fig. 1 were measured in different magnetic fields

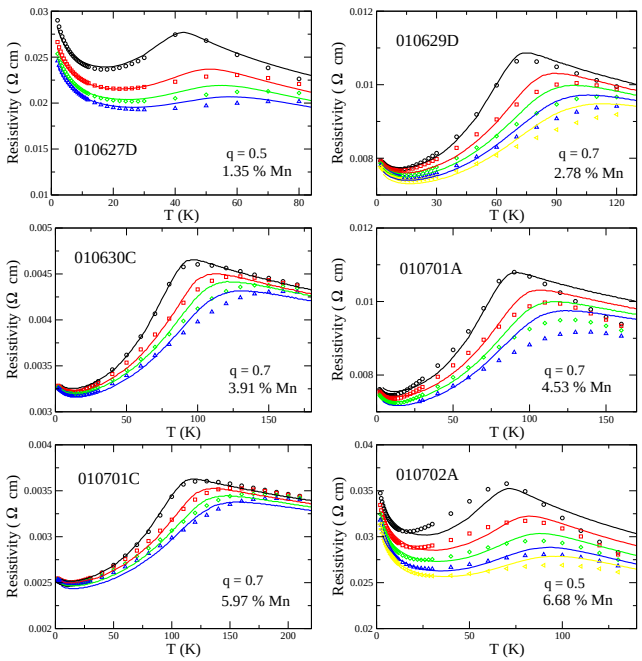

FIG. 1: Comparison between the experimental data and the theoretical results at magnetic fields H =0, 3, 6 and 9 T. Dots represent experimental data, solid lines are theoretical fits. Each figure indicates the value of q' that is used for theoretical fitting.

inside a commercial cryostat (Quantum Design PPMS), with the magnetic field normal to the sample plane. Details of the sample growth, materials characterization and annealing protocol were reported elsewhere [19].

In the present paper we shall use a slightly modified version of the scaling approach applied for disordered conductors.[20, 21, 22] In Ref. [18] we considered non-interacting electrons, where a scaling theory can be constructed in terms of the dimensionless conductance, g and a lengthscale at which electrons loose their coherence. In the presence of interactions, one also needs to introduce the dimensionless interaction parameters in the triplet and singlet channels, γ_t and γ_s, respectively. However, for $Ga_{1-x}Mn_xAs$ an important simplification occurs: $Ga_{1-x}Mn_xAs$ has a very large intrinsic spin-orbit gap, $\Delta_{so} \sim 4000$K. Furthermore, in the ferromagnetic phase and in the vicinity of the Curie temperature, T_C, the almost classical $S = 5/2$ spins of the Mn ions fluctuate slowly in time, and therefore, at the time scales and temperatures of interest, time reversal is also locally broken, even in the paramagnetic phase. As a result, as we showed already in Ref. 18, $Ga_{1-x}Mn_xAs$ belongs to the *unitary* class.[23] Then the γ_t plays no role, and γ_s can also be set to $\gamma_s = 1$.[21, 22] As a result, the scaling of the dimensionless conductance is described by a scaling equation,

$$\frac{d \ln g}{dx} = \beta(g) \ , \qquad (1)$$

where $x = \ln(\xi)$ is a scaling variable with $\xi = \xi(T)$ a lengthscale, and due to the simplifications above, the β-function depends only on g itself [21, 24]. In three dimensions, there is a metal-insulator transition characterized by $\beta(g_C) = 0$, with g_C the critical conductance. For $g > g_C$ and $\beta(g_C) > 0$, the conductor is metallic, and the dimensionless conductance increases with increasing system size, while $\beta(g < g_C) < 0$, and one recovers an insulating state.

Would we know the β-function, we could compute the resistivity as follows: Suppose we know the typical dimensionless conductance g_0 at an energy scale T_0 and at the corresponding microscopic length scale, $\xi_0 = \xi(T_0)$. Then the resistivity of a large three-dimensional conductor can be computed by integrating the scaling equation up to a length scale $x = \ln(\xi(T)/\xi_0)$ and cutting the system to small cubes of size $\xi = \xi(T)$ to give

$$\varrho(T, H) = \frac{h}{e^2} \frac{\xi(T)\, g_C}{g(\xi(T), g_0/g_C)} \ . \qquad (2)$$

Note that here we compute the typical conductance. Corrections due to universal conductance fluctuations can give a singular contribution, [8]. However, these corrections are very small for magnets with a short mean free path such as $Ga_{1-x}Mn_xAs$, and therefore they cannot explain the resistivity anomaly in $Ga_{1-x}Mn_xAs$ [8].

To compute $g(\xi)$ we need to know the beta-function. Unfortunately, while this can and has been determined numerically for a non-interacting unitary system [18], it is not known for interacting electrons. Here we shall use its asymptotic form on the metallic side, $\beta(g) = 1 - g_C^{(0)}/g$, with $g_C^{(0)}$ the critical value of the conductance to lowest order in the epsilon expansion.

Furthermore, we need to know the connection between the scale $\xi(T)$ and the temperature T. This connection can be established by looking at the pole structure of the diffusion propagator, $\xi^2(T) \sim D(T)/(T\, z(T))$, and using the Einstein relation, $\sigma(T) \sim (e^2/h)\; N(0) D(T)$ together with Eq. (2) [21, 22]. However, since we are interested in the metallic regime, with a good approximation, we can neglect the energy-dependence of the factor $z(T)$ and set it to $z \equiv 1$. As a result, we have $N(0)\; \xi^3 \sim g(\xi)/T$ [21], which can equally be written as

$$\left(\frac{\xi}{\xi_0}\right)^3 = \frac{g(\xi/\xi_0, g_0)}{g_0}\frac{T_0}{T}\;, \qquad (3)$$

with T_0 the energy scale corresponding to the length scale ξ_0.

With the $\beta(g)$ and $\xi(T)$ at hand, we then only need one more ingredient, the microscopic resistivity, $g_0 = g_0(h,t)$, with $t = T/T_C$ and $h = g\,\mu_B\, H\, S/T_C$ denoting the dimensionless temperature and magnetic field, respectively. In the following, we shall assume that g_0 depends exclusively on the magnetization m of the sample,

$$g_0\,(t,h) = g_0(m(t,h)) \approx \tilde{g}_0\,\left(1 + q\, m^2\,(t,h)\right) \qquad (4)$$

where $\tilde{g}_0$ is the conductance of the unpolarized system. This approximation is well justified within a mean-field description of the scattering on spin disorder [18], but it also emerges quite naturally for mechanisms where the primary role of the local magnetic moments is just to polarize the charge carriers and indirectly influence their scattering rate [25]. In the present formalism, however, the precise microscopic origin of the m-dependence of g_0 is of secondary importance.

The quadratic term in Eq. (4) is the leading contribution allowed by time reversal symmetry, and provides a very good approximation even in the extreme case of the infinite coupling disordered Kondo lattice [18]. From the fits we found that the parameter $q \approx 0.5 \div 0.7$ for all samples, in rough agreement with the results of [25].

In principle, we could now use the experimental $m(t,h)$ curves to compute the magnetoresistance of a sample. Unfortunately, experimentally, at large magnetic fields we could not separate the magnetization of the $Ga_{1-x}Mn_xAs$ film and that of the paramagnetic substrate, and only magnetization curves at small magnetic fields (H = 50 Oe) were available. Therefore, instead of using the experimental data, we determined the magnetization $m(t,h)$ in a finite field by performing simulations for a diluted spin system. To simulate $Ga_{1-x}Mn_xAs$ we placed magnetic ions with a given concentration at random positions of an FCC lattice following the procedure of Ref.[26], We assumed an RKKY interaction between the Mn spins and computed the magnetization curves $m(t,h)$ by performing a Monte Carlo simulation. This procedure reproduces the $m(t,h)$ curves, which turn out to be almost concentration-independent (apart from the limit of very small Mn concentrations), and fit very nicely the experimentally measured magnetization for $h = 0$ too (see Fig. 2). Although these magnetization curves look rather similar to the ones obtained from a simple mean field approximation, T_C is suppressed by a factor ~ 2 compared to the mean field Curie temperature due to thermal fluctuations. These thermal fluctuations being very sensitive to the magnetic field, the magnetization induced by an external field of ~ 1 T at $T = T_C$ was about a factor ~ 1.5 larger than the mean field estimate. This increased response was important to obtain the correct position of the resistivity maximum in finite magnetic fields. Note, however, that the the curves $m(t,h)$ obtained this way have no free fitting parameters.

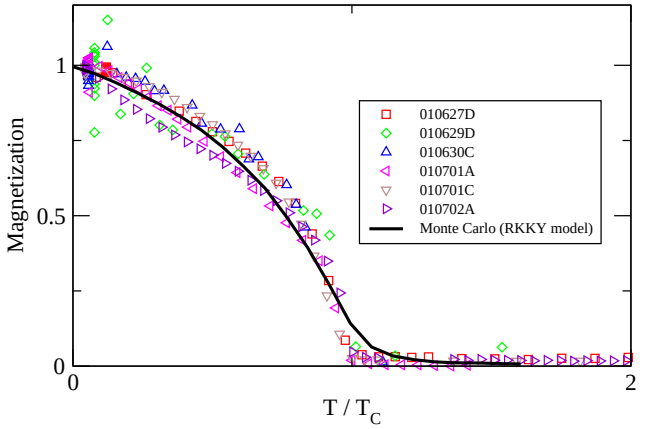

FIG. 2: Temperature dependence of the normalized magnetization in an external (H=50 Oe) in-plane magnetic field. For each sample the we rescaled T by the corresponding T_C. The solid line is the result of a Monte Carlo calculation for a diluted magnetic semiconductor $Ga_{1-x}Mn_xAs$ by assuming a RKKY interaction between the localized Mn spins.

We now have all ingredients to compute the magnetoresistance. The temperature and magnetic field-dependence of the resistivity then originates from the temperature and magnetic field-dependence of the microscopic conductance g_0 and that of the scale ξ: The correlation length ξ becomes larger as $T \to 0$, and therefore the resistivity increases. This results ultimately in the low-temperature upturn of the resistivity and is also responsible for the upturn of the resistivity above T_C. At very low temperatures this results in a $\sim \sqrt{T}$-dependence [27]. Entering the ferromagnetic phase, or polarizing the Mn moments with an external field, on the other hand, increases g_0, and hence decreases the resistivity. It is the

TABLE I: Characteristic parameters of the samples analyzed: x is the Mn concentration, $\varrho(T_C)$ is the resistivity at $T_0 \equiv T_C$, and k_F stands for the Fermi momentum obtained by assuming a compensation of 50%. We computed $k_F l$ from the Drude formula. We also show the Fermi wavelength λ_F of the non-magnetic system, the fitted correlation length at $T_0 = T_C$ [ξ_0] and at $1K$ [$\xi(1K)$], respectively, the dephasing length $\xi^{\rm Drude}(T_C)$ obtained using the Drude estimate, as described in the text. Finally we list the fitted values of $\tilde{g}_0/g_C$.

Sample number	x (%)	T_C (K)	$\varrho(T_C)$ $\Omega\, cm$	$k_F\, l(T_C)$	$\lambda_F(T_C)$ (nm)	$\xi^{\rm Drude}(T_C)$ (nm)	$\xi^{\rm fit}(1K)$ (nm)	$\xi^{\rm fit}(T_C)$ (nm)	$\tilde{g}_0/g_C$
010627D	1.35	42	27×10^{-3}	0.24	3.34	3.59	54.43	8.4	1.35
010629D	2.78	65	10×10^{-3}	0.51	2.64	4.38	70.94	8.8	2.85
010630C	3.91	90	4.5×10^{-3}	1.01	2.35	5.29	96.76	10.2	4.16
010701A	4.53	85	11×10^{-3}	0.39	2.24	3.43	94.03	10.2	4.01
010701C	5.87	110	3.6×10^{-3}	1.10	2.04	4.95	100.68	9.6	4.76
010702A	6.68	70	35×10^{-3}	0.10	1.97	1.82	73.62	8.8	1.70

competition of these two effects that yields the resistivity anomaly at T_C.

Eqs. (2) and (4) together with Eq. (3) provide a full theoretical description of the magneto-resistance in terms of three parameters for every sample, ξ_0, $\tilde{g}_0/g_C$, and the phenomenological parameter q. Fig. 1 shows the best fits obtained in this way for six different samples. For all samples we defined ξ_0 and g_0 as the scale and dimensionless conductance at $T_0 \equiv T_C$. The parameters of the samples are summarized in Table I. Note that the position, the shift and the amplitude of both the resistivity maxima at T_C as well as that of the low-temperature anomaly work out very nicely for these samples. This proves indirectly, that both anomalies are related to the vicinity of the metal-insulator transition.

In Table I we also enumerate the values of $k_F l$ we obtain from the resistivity by assuming a valence hole band of effective mass $m^* = 0.45\ m_e$ and a compensation of 50%. Clearly, the values obtained in this way are inconsistent with a weakly disordered free electron picture, but could naturally be explained through the presence of an impurity band with an enhanced carrier mass.[12].

Importantly, the fitted values of ξ are smaller than the thickness of the films, $W \approx 123$ nm even at $T = 1K$, and they are in good agreement with the values obtained in Refs. [28]. These samples are thus three-dimensional from the point of view of conductance properties down to these temperatures. Also, ξ remains larger than the typical Mn-Mn separation $\sim$ 1nm over the whole range of temperatures, thereby justifying the scaling approach used here. In Table I we also included the theoretical estimate of $\xi^{\rm Drude}(T) = \sqrt{D_{\rm Drude}/T}$ where we estimated $D_{\rm Drude}$ by using the Drude formula and the density of states of a parabolic valence band with a renormalized mass. The values obtained in this way do not depend too much on the specific sample, and, apart from an overall factor, are in rough agreement with the values extracted from the fits. This is somewhat surprising in view of the extremely small values of $k_F l$.

In conclusion, we presented here a systematic study of the resistivity of various $Ga_{1-x}Mn_xAs$ samples. We argued that even the annealed samples are very close to the localization transition and showed that the complete magnetic field dependence of the resistivity anomaly at T_C as well as the low temperature upturn of the resistivity can be *quantitatively* described in terms of a perturbative scaling theory of localization, combined with Monte Carlo simulations and a reasonable choice of materials parameters.

We would like to thank J.K. Furdyna, X. Liu, A. H. MacDonald, J. Sinova, and T. Jungwirth, but especially A.M. Finkelstein for helpful discussions. This research has been supported by Hungarian grants OTKA Nos. NF061726, T046267, and T046303, Romanian grant CNCSIS 2006/1/97 and CNCSIS 2007/1/780. BLS, PS, and NS were supported by the NSF and the NNIN. B.J. was supported by NSF-NIRT award No. ECS-0609249, as well as the US. Department of Energy, Basic Energy Sciences. under Contract No. W-31-109-ENG-38.

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